

# On theories of enhanced CP violation in $B_{s,d}$ meson mixing.

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The DØ collaboration has measured a deviation from the standard model (SM) prediction in the like sign dimuon asymmetry in semileptonic  $b$  decay with a significance of  $3.2\sigma$ . We discuss how minimal flavour violating (MFV) models with multiple scalar representations can lead to this deviation through tree level exchanges of new MFV scalars. We review how the two scalar doublet model can accommodate this result and discuss some of its phenomenology. Limits on electric dipole moments suggest that in this model the coupling of the charged scalar to the right handed  $u$ -type quarks is suppressed while its coupling to the  $d$ -type right handed quarks must be enhanced. We construct an extension of the MFV two scalar doublet model where this occurs naturally.

## INTRODUCTION

The DØ collaboration has reported a  $3.2\sigma$  deviation from the standard model (SM) prediction of the like sign dimuon asymmetry in semileptonic  $b$  decay [1]. This observation joins past anomalous measurements of  $B_s \rightarrow J/\psi \phi$  and  $B^- \rightarrow \tau \nu$  decays that can be interpreted as a pattern of deviations consistent with new physics contributing a new phase in  $B_{s,d}$  mixing (for a recent global fit and discussion see [2]).<sup>1</sup>

If the explanation of the like sign dimuon asymmetry measurement and these correlated deviations are not statistical fluctuations, then new physics interpretations of this pattern are of interest. General operator analyses have been carried out [10–12] and indicate that operators induced by scalar exchange with unenhanced Yukawa couplings and order one parameters in the potential (i.e. order one Wilson coefficients) the mass scale suppressing the operators of interest is a few hundred GeV.

Such a low mass scale is challenging to reconcile with known constraints from flavour physics unless minimal flavour violation (MFV) [13–15] is imposed. New physics (NP) models with MFV have the quark flavor group  $SU(3)_{U_R} \times SU(3)_{D_R} \times SU(3)_{Q_L}$  only broken by the Yukawa couplings. However this scenario does allow new phases and so provides a framework for explaining the anomalies mentioned above without giving rise to flavor changing neutral current effects that are in conflict with experiment.

In this paper we discuss scalar models with MFV that can explain these anomalies. We first review how tree level exchanges of a neutral complex scalar in a simple two scalar doublet model can lead to enhanced CP violation in the  $B_q$  meson system and discuss the phenomenology of this model. We then show that limits on electric dipole moments suggest that the coupling of the charged scalar to the right handed  $u$ -type quarks is suppressed while its coupling to the  $d$ -type right handed quarks must be enhanced to be consistent with the data. We construct an extension of the MFV two scalar doublet model where this occurs naturally<sup>2</sup>.

## SET UP

We will utilize the recent fit of [2] to determine the new contribution to  $B_q - \bar{B}_q$  mixing (here  $q = s, d$ ). This fit is consistent in its conclusions with an earlier analysis [10]. The DØ result ( $a_{SL}^b$ ) and the SM prediction [2] ( $A_{SL}^b$ ) are given by

$$a_{SL}^b = \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}},$$

$$= -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}, \quad (1)$$

$$A_{SL}^b = (-3.10_{-0.98}^{+0.83}) \times 10^{-4}. \quad (2)$$

where the number of  $X b \bar{b} \rightarrow \mu^+ \mu^+ Y$  events is given by  $N_b^{++}$  for example. The quoted  $a_{SL}^b$  is a combination of the the asymmetry in each  $B_q$ , denoted  $a_{SL}^{bq}$ . Each of these contributions to  $a_{SL}^b$  can be expressed in terms of the mass and width differences ( $M_{12}, \Gamma_{12}$ ) of the  $B_q$  meson eigenstates and the CP phase difference between

<sup>1</sup> The observed  $2.6\sigma$  deviation from the standard model (SM) expectation [2] in the averaged measurements of  $B^- \rightarrow \tau \nu$  performed at Belle and Babar [3–6] correlates correctly with a new physics (NP) contribution of a phase to  $B_d$  with a sign consistent with the NP phase implied by the DØ dimuon measurement. Such a NP phase also correlates with the expectation of a shift in  $\sin 2\beta$  extracted from  $B_s \rightarrow J/\psi \phi$  compared to the SM expectation [7, 8] and extractions from measurements in  $B_s \rightarrow J/\psi K_s$ . Such a consistent deviation is also observed, its statistical significance is  $2.1\sigma$ . Also see [9] for a discussion on the evidence for a NP phase in  $B_d$  and  $B_s$  meson mixing.

<sup>2</sup> Of course models with scalar doublets that are not supersymmetric suffer from the well know naturalness problem of keeping the doublets light compared to the Planck scale.

these quantities  $\phi_q$  as

$$a_{SL}^{bq} = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin \phi_q. \quad (3)$$

Naively one can effect the SM prediction through modifying  $M_{12}$  or  $\Gamma_{12}$  and both approaches have been explored in the literature. Modifying the decay width significantly [16, 17] as an explanation is problematic<sup>3</sup> and we will focus on MFV NP explanations that involve a NP contribution (that includes a new CP violating phase) to  $M_{12}^q$ .

The effect of NP on  $B_s$  and  $B_d$  mass mixing can be parametrized by two real parameters,  $h_q > 0$  and  $\sigma_q$  by writing

$$M_{12}^q = (M_{12}^q)^{\text{SM}} + (M_{12}^q)^{\text{NP}}, \quad (4)$$

where the new physics contribution to the mass mixing is related to the standard model value of the mass mixing by,

$$(M_{12}^q)^{\text{NP}} = (M_{12}^q)^{\text{SM}} h_q e^{2i\sigma_q}. \quad (5)$$

The models we discuss have  $h_s = h_d$  and  $\sigma_s = \sigma_d$  which is generally expected in NP models that obey MFV.<sup>4</sup> This scenario is argued to be a better fit to the current data than the SM in [2], which is disfavoured with a p-value of  $3.1\sigma$ . In this case, the best fit values are  $h_q = 0.255$  and  $2\sigma_q = 180^\circ + 63.4^\circ$ . The best fit magnitude of the correction  $h_q$  is small but its phase is large.

For simplicity in this paper we treat perturbative QCD in the leading logarithmic approximation and evaluate the needed matrix elements of four quark operators using the vacuum insertion approximation at the bottom mass scale. At the  $t$ -quark mass scale, in the SM, the effective Hamiltonian for  $B_q - \bar{B}_q$  mixing is,

$$\mathcal{H}_q^{\text{SM}} = (V_{tq}^* V_{tb})^2 C^{\text{SM}}(m_t) \bar{b}_L^\alpha \gamma^\mu q_L^\alpha \bar{b}_L^\beta \gamma_\mu q_L^\beta, \quad (6)$$

<sup>3</sup> The decay width can be removed in the relation between measured quantities under the assumption of small CP violation in NP induced tree level decays of  $B_q$  [10, 18] and the anomalous measurements can still be fit to finding  $\sim 3\sigma$  evidence for a deviation from the SM [2, 10]. Also, an explanation of the like sign dimuon asymmetry through a NP contribution to  $|\Gamma_{12}^q|$  would not necessarily explain the anomalies in  $B_s \rightarrow J/\Psi \phi$  and  $B \rightarrow \tau \nu$  with the correct correlation.

<sup>4</sup> It has been proven in [27] that new CP violating effects can be larger in  $B_s$  than in  $B_d$  in nonlinear MFV. This observation has recently been explored in a general operator analysis [12] which showed that enhancements of CP violation in  $B_s$  mixing over  $B_d$  mixing by  $m_s/m_d$  requires contributions in the MFV expansion out to forth order in both the up and down Yukawas for operators induced by scalar exchange. The results on the neutron EDM using naive dimensional analysis (NDA) on page 4 disfavour order one down and up Yukawas with CP violating phases for these operators, so such contributions are expected to be very small. For alternative estimates of the relevant matrix element not using NDA see [19].

where  $\alpha$  and  $\beta$  are color indices and

$$C^{\text{SM}}(m_t) = \frac{G_F^2}{4\pi^2} M_W^2 S(m_t^2/M_W^2). \quad (7)$$

Here  $S(m_t^2/M_W^2) \simeq 2.35$  is a function of  $m_t^2/M_W^2$  that results from integrating out the top quark and  $W$ -bosons. Using

$$(M_{12}^q)^{\text{SM}} = \frac{\langle B_q | \mathcal{H}_q^{\text{SM}} | \bar{B}_q \rangle}{2m_{B_q}}, \quad (8)$$

we have after running down to the  $b$ -quark mass scale that,

$$(M_{12}^q)^{\text{SM}} = (V_{tq}^* V_{tb})^2 C^{\text{SM}}(m_t) \left( \frac{1}{3} \right) \eta f_{B_q}^2 m_{B_q}. \quad (9)$$

Here  $\eta \simeq 0.84$  is a QCD correction factor,  $C_q^{\text{SM}}(m_b) = \eta C_q^{\text{SM}}(m_t)$ .

The models for new physics we discuss generate the effective Hamiltonian at the top scale<sup>5</sup>

$$\mathcal{H}_q^{\text{NP}} \simeq (V_{tq}^* V_{tb})^2 C^{\text{NP}}(m_t) \bar{b}_R^\alpha q_L^\alpha \bar{b}_R^\beta q_L^\beta. \quad (10)$$

Running down from  $m_b$  operator mixing induces the analogous operator with color indices rearranged. However, its coefficient is very small and we neglect it resulting in the relation,  $C^{\text{NP}}(m_b) \simeq \eta' C^{\text{NP}}(m_t)$ , where  $\eta' \simeq 1.45$  [20]. Again using the vacuum insertion approximation at the  $b$ -quark mass scale we arrive at,

$$(M_{12}^q)^{\text{NP}} \simeq (V_{tq}^* V_{tb})^2 C^{\text{NP}}(m_t) \left( -\frac{5}{24} \right) \eta' f_{B_q}^2 m_{B_q}. \quad (11)$$

Comparing with Eq. (5)

$$h_q e^{2i\sigma_q} \simeq -\frac{5}{8} \left( \frac{C^{\text{NP}}(m_t)}{C^{\text{SM}}(m_t)} \right) \frac{\eta'}{\eta}. \quad (12)$$

## MINIMAL TWO SCALAR DOUBLET MODEL

We now discuss how the minimal two scalar doublet model with MFV can have enhanced CP violation in  $B_q$  mixing due to tree level exchange of neutral scalars.<sup>6</sup> We denote by  $H$  the doublet that gets a vacuum expectation value and by  $S$  the doublet that does not. The Lagrangian in the Yukawa sector is

$$\begin{aligned} \mathcal{L}_Y = & \bar{u}_R^i g_U^j Q_{Lj} H + \bar{d}_R^i g_D^j Q_{Lj} H^\dagger \\ & + \bar{u}_R^i Y_U^j Q_{Lj} S + \bar{d}_R^i Y_D^j Q_{Lj} S^\dagger + \text{h.c.} \end{aligned} \quad (13)$$

<sup>5</sup> For QCD running we don't distinguish between the top scale, weak scale and the mass scale of the new scalars we shall add.

<sup>6</sup> Previous analyses focused on scalar exchange to explain the like sign dimuon asymmetry include [21–23]. Also see [24, 25] for some phenomenological studies of models of this form.

where flavour indices  $i, j$  are shown and color and  $SU(2)_L$  indices have been suppressed. MFV asserts that any NP also has the quark flavour symmetry group only broken by insertions proportional to Yukawa matrices so that  $Y_U^j, Y_D^j$  are proportional to  $g_U^j, g_D^j$ . One can construct allowed NP terms by treating the Yukawa matrices as spurion fields that transform under flavour rotations as,

$$g_U \rightarrow V_U g_U V_Q^\dagger, \quad g_D \rightarrow V_D g_D V_Q^\dagger, \quad (14)$$

where  $V_U$  is an element of  $SU(3)_{U_R}$ ,  $V_D$  is an element of  $SU(3)_{D_R}$ , and  $V_Q$  is an element of  $SU(3)_{Q_L}$ , i.e., the Yukawa matrices transform as  $g_U \sim (\mathbf{3}, \mathbf{1}, \mathbf{\bar{3}})$  and  $g_D \sim (\mathbf{1}, \mathbf{3}, \mathbf{\bar{3}})$  under the flavour group. MFV can be formulated up to linear order in top Yukawa insertions, or extended to a nonlinear representation of the symmetry [26, 27]. For enhanced CP violation in  $B_q$  mixing we are interested in a nonlinear realization of MFV. It is sufficient to only expand to next order in insertions of  $g_U$  so that

$$\begin{aligned} Y_U^j &= \eta_U g_U^j + \eta'_U g_U^j [(g_U^\dagger)^k_l (g_U)^l_i] + \dots, \\ Y_D^j &= \eta_D g_D^j + \eta'_D g_D^j [(g_U^\dagger)^k_l (g_U)^l_i] + \dots. \end{aligned} \quad (15)$$

We decompose the second scalar doublet as

$$S = \begin{pmatrix} S^+ \\ S^0 \end{pmatrix}, \quad (16)$$

where  $S^0 = (S_R^0 + iS_I^0)/\sqrt{2}$ .

The scalar potential is

$$\begin{aligned} V &= \frac{\lambda}{4} \left( H^{\dagger i} H_i - \frac{v^2}{2} \right)^2 + m_1^2 (S^{\dagger i} S_i), \\ &+ (m_2^2 H^{\dagger i} S_i + \text{h.c.}) + \lambda_1 (H^{\dagger i} H_i) (S^{\dagger j} S_j), \\ &+ \lambda_2 (H^{\dagger i} H_j) (S^{\dagger j} S_i) + [\lambda_3 H^{\dagger i} H^{\dagger j} S_i S_j + \text{h.c.}], \\ &+ [\lambda_4 H^{\dagger i} S^{\dagger j} S_i S_j + \lambda_5 S^{\dagger i} H^{\dagger j} H_i H_j + \text{h.c.}], \\ &+ \lambda_6 (S^{\dagger i} S_i)^2. \end{aligned} \quad (17)$$

where  $i, j$  are  $SU(2)$  indices. Here  $v \simeq 246\text{GeV}$  is the vacuum expectation value (vev) of the Higgs. Since we adopted the convention that the doublet  $S$  does not get a vev the parameters  $m_2^2$  and  $\lambda_5$  are related by,

$$m_2^2 + \lambda_5^* \frac{v^2}{2} = 0. \quad (18)$$

The spectrum of neutral real scalars consists of the Higgs scalar  $h$  and  $S_R^0$  and  $S_I^0$ . However, these are not mass eigenstates. In the  $(h, S_R^0, S_I^0)$  basis, the neutral mass squared matrix  $\mathcal{M}^2$ , where  $\lambda_3$  is chosen real and positive, is

$$\mathcal{M}^2 = \begin{pmatrix} m_h^2 & \lambda_5^R v^2 & \lambda_5^I v^2 \\ \lambda_5^R v^2 & m_S^2 + \lambda_3 v^2 & 0 \\ \lambda_5^I v^2 & 0 & m_S^2 - \lambda_3^2 \end{pmatrix}. \quad (19)$$

Where  $m_S^2 = m_1^2 + (\lambda_1 + \lambda_2)v^2/2$ . Within the convention that  $\lambda_3$  is real, the couplings  $\eta_U, \eta'_U, \eta_D$  and  $\eta'_D$  and  $\lambda_5 = \lambda_R^R + i\lambda_5^I$  are in general complex. The mass eigenstate scalars  $N_j^0$  with mass  $m_j$  are related to  $h, S_R^0, S_I^0$  by the orthogonal transformations

$$\begin{aligned} h &= \sum_j O_{hj} N_j, \\ S_R &= \sum_j O_{Rj} N_j, \quad S_I = \sum_j O_{Ij} N_j. \end{aligned} \quad (20)$$

We find the CP violating NP contribution to  $B_q - \bar{B}_q$  mixing from neutral scalar exchange is

$$C^{\text{NP}}(m_t) = \left( \sqrt{2} \eta'_D m_b/v \right)^2 \left( F \left( \sqrt{2} m_t/v \right) \right)^2 \frac{\Delta}{2}, \quad (21)$$

where  $F(x) = x^2 + \dots$ , and

$$\Delta = \sum_j \frac{(O_{Rj} + i O_{Ij})^2}{m_j^2}. \quad (22)$$

For the rest of this paper we truncate the expansions in  $\sqrt{2} m_t/v$  at the leading non trivial term. So, for example, in Eq. (23) we use  $F(x) = x^2$ .

For simplicity we now focus on the case where  $\lambda_5 = 0$ . Then  $h, S_R^0$  and  $S_I^0$  are mass eigenstates and the term in the potential proportional to  $\lambda_3$  is of interest as it leads to the mass splitting between the real neutral fields given by  $m_R^2 - m_I^2 = 2\lambda_3 v^2$ . When this term is non vanishing<sup>7</sup>, the tree level exchange of  $S_{R/I}$  generates a CP violating NP contribution to  $B_q - \bar{B}_q$  mixing and

$$C^{\text{NP}}(m_t) = (\eta'_D)^2 \left( \frac{\sqrt{2} m_t}{v} \right)^4 \left( \frac{\lambda_3 m_b^2}{m_S^4 - \lambda_3^2 v^4} \right). \quad (23)$$

In the above equation, the bottom quark mass  $m_b \simeq 2.93\text{ GeV}$  is evaluated at the top quark mass scale and  $m_{S_{R/I}}^2 = m_S^2 \pm \lambda_3 v^2$ .

Using Eq. (12) the mass scale of the new scalars is given by

$$m_S^4 \simeq \frac{20 \pi^2 \lambda_3 |\eta'_D|^2 \eta' m_b^2 m_t^4}{h_q \eta M_W^2 S(m_t^2/m_W^2)} + \lambda_3^2 v^4. \quad (24)$$

Using the best fit value  $h_q = 0.255$  [2] we find that

$$m_S^4 \simeq (154\text{ GeV})^4 |\eta'_D|^2 \lambda_3 + (246\text{ GeV})^4 \lambda_3^2. \quad (25)$$

Then for example with a value  $|\eta'_D| = 5$  and  $\lambda_3 = 1$  the scalar mass scale is  $m_S \simeq 360\text{ GeV}$ . As the mass

<sup>7</sup> Note that imposing custodial symmetry on the potential does not force  $\lambda_3 \rightarrow 0$ . Custodial symmetry violation is a measure of the total mass splitting  $(m_R^2 - m_\pm^2)(m_I^2 - m_\pm^2) \propto (\lambda_2^2 - (2\lambda_3)^2) v^4$  in terms of the potential given in Eq. (17).

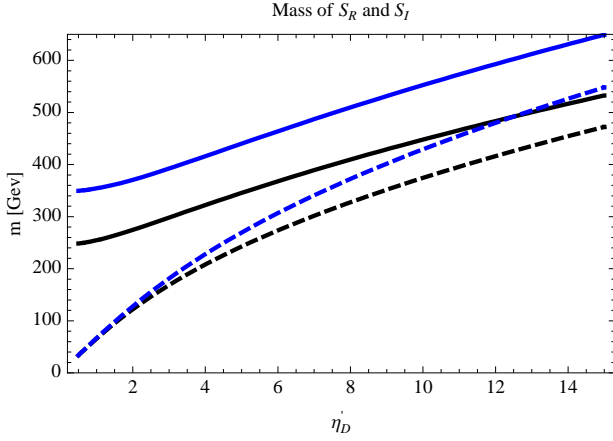


FIG. 1: Mass of  $S_R$  (solid) and  $S_I$  (dashed) as a function of  $\eta'_D$  for fixed  $\lambda_3$ . The upper (blue curves) are for  $\lambda_3 = 1$  the lower (black) curves are for  $\lambda_3 = 0.5$ .

splitting is significant we show in Fig. 1 the masses of the neutral scalars  $S_R, S_I$  as a function of  $\eta'_D$ . Moderate enhancements of  $\eta'_D$  avoid a light neutral state.

We have checked that the mass scale  $m_S$  required for  $B_q - \bar{B}_q$  mixing is compatible with the constraints from  $K - \bar{K}$  mixing. This compatibility is due to MFV, which causes the ratio of the relevant Wilson coefficients to scale as  $m_s^2/m_b^2 |V_{td}/V_{tb}|^2$ .

Next we derive constraints on  $\eta_U \eta_D$  that come from one loop Feynman diagrams with charged  $S$  scalar exchange. We will show that limits on electric dipole moments imply that  $|\text{Im}[\eta_U \eta_D]| \lesssim 10^{-1}$ . Note that writing this as a constraint just on  $\eta_U \eta_D$  depends on truncating a function of  $\sqrt{2}m_t/v$  at leading order. We also examine the constraint on  $\text{Re}[\eta_U \eta_D]$  coming from experimental data on weak radiative  $B$  decay.

#### Neutron Electric Dipole Moment

The large CP violating phases needed in this two scalar doublet model contribute to other CP violating observables. Notable among them are electric dipole moments (EDM's). We will restrict our discussion here to the dominant contribution that is not suppressed by small quark masses when naive dimensional analysis (NDA) [28] is used. It comes about through the colour electric dipole moment of the  $b$  quark [29, 30] due to the effective Hamiltonian

$$\delta\mathcal{H}_{bg} = C_{gb} g_3 m_b \bar{b} \sigma_{\mu\nu} T_a G_{\lambda\sigma}^a \epsilon^{\mu\nu\lambda\sigma} b, \quad (26)$$

inducing the dimension six CP violating operator

$$O_G = g_3^3 f_{\alpha\beta\gamma} \epsilon^{\mu\nu\lambda\sigma} G_{\alpha\mu\rho} G_{\beta\nu}^\rho G_{\gamma\lambda\sigma}, \quad (27)$$

of Weinberg [31] when the  $b$  quark is integrated out. Our discussion will largely parallel the discussion of [32]. As  $S$

couples both to the up and down type quarks it induces a one loop contribution to the effective Hamiltonian above with

$$C_{gb}(m_S) = \frac{-\text{Im}[\eta_U^* \eta_D^*]}{64\pi^2 m_{S\pm}^2} \left( \frac{\sqrt{2}m_t}{v} \right)^2 f(m_t^2/m_{S\pm}^2), \quad (28)$$

where

$$f(x) = \frac{\log x}{(x-1)^3} + \frac{x-3}{2(x-1)^2}. \quad (29)$$

Running to  $\mu \sim m_b$  using [29, 30] and estimating the matrix element of the operator with NDA<sup>8</sup> gives in e-cm units

$$d_n \sim 2 \text{Im}[\eta_U^* \eta_D^*] f(m_t^2/m_{S\pm}^2) \left( \frac{1 \text{ TeV}}{m_{S\pm}} \right)^2 10^{-26}. \quad (30)$$

This is a significantly larger effect on EDM's than quoted in the general operator analysis [11] examining the effects of four Fermi operators on EDM's as this contribution is not suppressed by small mixing angles or light quark masses. For  $m_{S\pm} = 360$  GeV the neutron EDM experimental bound of  $d_n < 2.9 \times 10^{-26}$  e-cm implies that  $|\text{Im}[\eta_U^* \eta_D^*]| < 0.26$ . We plot the allowed  $|\text{Im}[\eta_U^* \eta_D^*]|$  as a function of mass for this NDA estimate in Fig. 2.

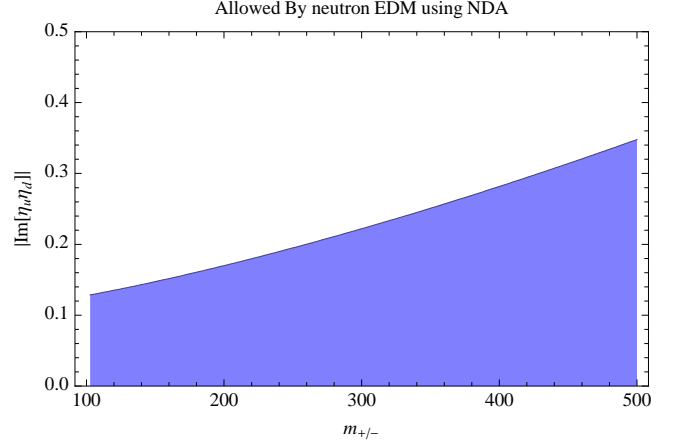


FIG. 2: Allowed  $|\text{Im}[\eta_U^* \eta_D^*]|$  as a function of the charged scalar mass.

This suggests that  $|\text{Im}[\eta_U^* \eta_D^*]|$  (and the sum of the effect of all other cross terms such as  $|\text{Im}[\eta_U^* \eta_D^*]|$  etc.) is also small. However, given the uncertainties from hadronic matrix elements and given the fact that the parameters that enter the contribution to EDM's are not identical to those in  $B_q - \bar{B}_q$  mixing it is difficult to draw precise conclusions on the parameters in the model that are important for mixing.

<sup>8</sup> We use method (a) of [32] with  $\alpha_s(\mu = 1 \text{ GeV}) \sim 4\pi$ .

### $B \rightarrow X_s \gamma$ Constraints

Of course the two scalar doublet model also gives new contributions to quantities that are not CP violating. Here we briefly review the constraints on this model from  $B \rightarrow X_s \gamma$  with these assumptions. The extra term in the effective Hamiltonian arises from charged scalar exchange and has the form,

$$\delta \mathcal{H}_{\bar{B} \rightarrow X_s \gamma} = [V_{ts}^* V_{tb}] C_\gamma \left( \frac{e m_b}{16 \pi^2} \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R \right), \quad (31)$$

where  $e < 0$  is the electric charge. The Wilson coefficient is given by

$$C_\gamma = \eta_U^* \eta_D^* \left( \frac{2 m_t^2}{v^2} \right) \frac{f_\gamma(m_t^2/m_{S^\pm}^2)}{3 m_{S^\pm}^2}, \quad (32)$$

with

$$f_\gamma(x) = \frac{1}{4} \left( \frac{1 + 2x \log x - x^2}{(1-x)^3} \right) - \left( \frac{1 + \log x - x}{(1-x)^2} \right). \quad (33)$$

This operator's contribution to the measured branching fraction  $\text{BR}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}$  is known [33]

$$\frac{\text{BR}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}}{10^{-4}} = 3.15 \pm 0.23 - 4.0 v^2 C_\gamma.$$

This constraint includes the effect of running this operator down to the scale  $m_b$ . Comparing to the world experimental average [34] we obtain a  $1\sigma$  bound on the parameters of the form

$$-0.17 < \text{Re}[\eta_U^* \eta_D^*] f_\gamma(m_t^2/m_{S^\pm}^2) \frac{m_t^2}{3 m_{S^\pm}^2} < 0.07. \quad (34)$$

For  $m_{S^\pm} = 360 \text{ GeV}$  we find  $-1.7 < \text{Re}[\eta_U^* \eta_D^*] < 0.7$ . Although this constraint is weak it is interesting that EDM's constrain  $\text{Im}[\eta_U^* \eta_D^*]$  while  $B \rightarrow X_s \gamma$  constrains  $\text{Re}[\eta_U^* \eta_D^*]$ .

### Collider Physics: Two Scalar Doublet Model

Pairs of  $S$  particles can be produced through the tree level exchange of vector bosons produced through  $q \bar{q}$  initial states in the case of the Tevatron and LHC and  $e^+ e^-$  in the case of LEP II.

From LEP II a bound on the mass scale of the new scalar doublet is obtained as no anomalous two and four jet events were seen when operating at  $\sqrt{s} = 209 \text{ GeV}$  where  $0.1 \text{ fb}^{-1}$  of integrated luminosity was collected. The relevant cross sections in this case are given in [35] and the masses are bound to be

$$m_{S^\pm} \gtrsim 105 \text{ GeV}, \quad m_{S_R^0} + m_{S_I^0} \gtrsim 209 \text{ GeV}. \quad (35)$$

We plot the allowed  $m_S, \lambda_3$  that satisfy this second bound for the minimal two scalar doublet model in Fig.3. We

have also performed an electroweak precision data fit. For scalar masses  $\sim 100 \text{ GeV}$  the constraints are weak. The allowed mass splitting in this model is  $|m_I - m_\pm| \lesssim 200 \text{ GeV}$  using the 95%CL region.

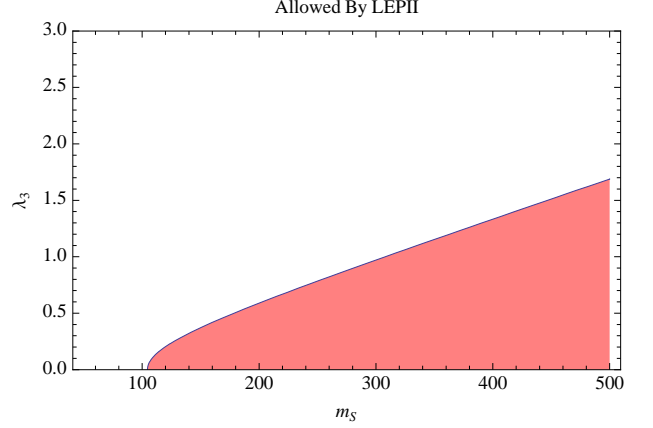


FIG. 3: The allowed range of parameters  $m_S, \lambda_3$  in the two scalar doublet model considering LEP II direct production bounds.

A light mass of  $S_I^0$  is allowed as these states must be produced in pairs through vector boson exchange and  $S_R^0$  can be heavy. However, a single neutral scalar particle can be produced at the Tevatron in association with a charged scalar through  $W^\pm$  exchange. The partonic production cross section for producing  $S^\pm S_I^0$  or  $S^\pm S_R^0$  (when the width is neglected) is

$$\sigma = \frac{(p^2/s)^{3/2}}{s_W^4} \left( \frac{\pi \alpha_e^2(M_Z)}{6 s} \right) \left| 1 - \frac{M_W^2}{s} \right|^{-2}, \quad (36)$$

where  $p$  is the center of mass momentum of one of the produced particles and  $s$  is the partonic center of mass energy squared. We scan over the parameter space allowed by LEP II using this formula for the Tevatron production cross section (with MSTW 2008 PDF's [36]) where the  $W^\pm$  is produced off the valence  $u, d$  quarks. The renormalization scale in what follows is always varied between  $m_S/2$  and  $2 m_S$ . The cross sections as a function of mass are shown in Fig. 4. The Tevatron can potentially constraint some of the allowed parameter space. Search strategies for pair production through weak boson fusion of charged  $S^\pm$  particles that decay into  $t \bar{b} b t$  are also somewhat promising. In this case the production cross section for  $m_S \sim 200 \text{ GeV}$  is  $\sigma \sim 1 \text{ fb}$  with a signal of two  $b$  jets and two  $t$  jets is produced in association with tagging light quark jets at large  $p_T$ .

At the LHC, production through the tree level exchange of a vector boson is no longer dominated by  $W^\pm$  exchange. The cross sections for the pair production of scalars are all similar in their dependence on  $\lambda_3$  and as a function of  $m_S$ . We show  $\sigma(pp \rightarrow W^\pm \rightarrow S^\pm S_{R/I})$  for  $\sqrt{s} = 7 \text{ TeV}$  in Fig. 5.

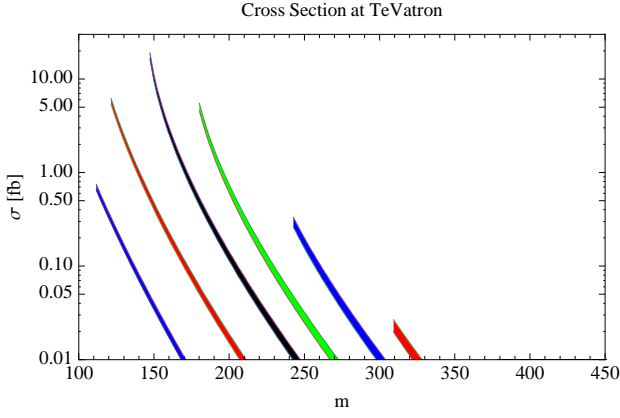


FIG. 4: The cross section  $\sigma(S^\pm S_I) + \sigma(S^\pm S_R)$  as a function of  $m_S$  for  $\lambda_3 = (0.1, 0.2, 0.35, 0.5, 0.75, 1)$  going left to right. Here we have also imposed custodial symmetry on the potential  $\lambda_2 = \pm 2\lambda_3$  for simplicity in the parameter scans.

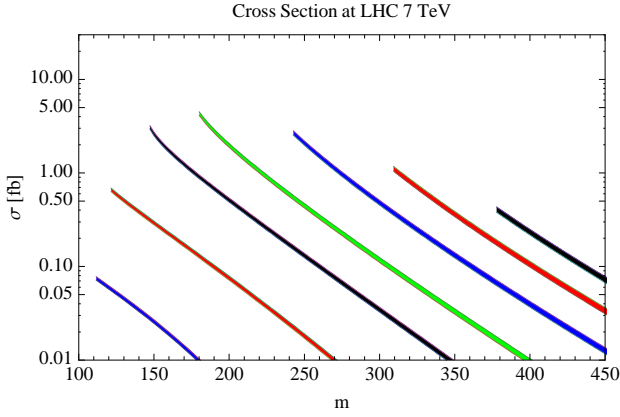


FIG. 5: The cross section  $\sigma(S^\pm S_I) + \sigma(S^\pm S_R)$  as a function of  $m_S$  for  $\lambda_3 = (0.1, 0.2, 0.35, 0.5, 0.75, 1, 1.25)$  going left to right. Here we have also imposed custodial symmetry on the potential  $\lambda_2 = \pm 2\lambda_3$  for simplicity in the parameter scans. The other production cross sections through  $Z^*, \gamma^*$  are similar.

Although these vector boson exchange cross sections for LHC are small, potentially observable signals at LHC do exist when  $\eta_D$  is larger than one, and  $S_{R/I}$  is made with large logarithms associated with collinear gluon splitting [37] and small  $p_T$  of the spectator  $b$  quarks. The cross-section for the production of the lightest state  $b\bar{b} \rightarrow S_I^0$  at leading log takes the form [38]

$$\sigma(b\bar{b}S_I^0) \simeq \frac{|\eta_D|^2 \pi}{3s} \left( \frac{m_b^2}{v^2} \right) \int_{\frac{m_{S_I}^2}{s}}^1 \frac{dx}{x} b(x, \mu) \bar{b}\left(\frac{m_{S_I}^2}{xs}, \mu\right), \quad (37)$$

where  $b(x, \mu)$  and  $\bar{b}(x, \mu)$  are the  $b$  quark and anti-quark PDFs respectively. The large logs from collinear gluon splitting are summed into the parton distribution functions by choosing  $\mu \sim m_S$ . When we let  $\eta_D = \sqrt{m_S/(154 \text{ GeV})}$  and choose  $\lambda_3 = 1$  the production cross sections for the LHC are given by Fig.6. This pro-

duction mechanism must compete with the large  $b$  production background from QCD. However, we note that this signal has a distinct feature in its reconstruction of a resonance in the highest  $p_T$   $b$  quark pair with a larger percentage of its total number of events at high  $p_T$  and small rapidity than the SM background, which has an approximate Rutherford scattering angular dependence in its production of  $b$  quarks.

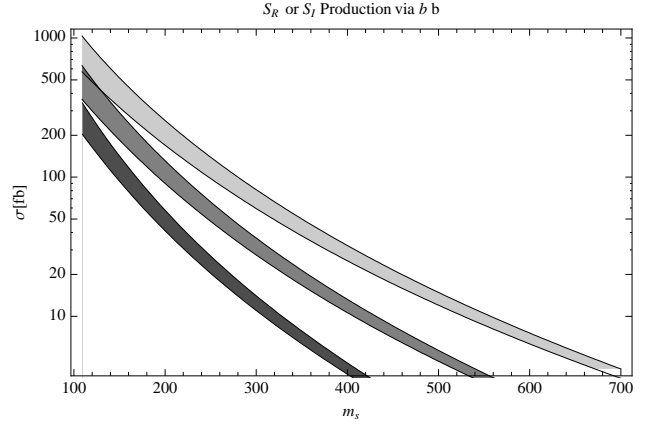


FIG. 6: The cross section  $\sigma(pp \rightarrow b\bar{b}S_I^0)$  for  $S_I^0$  produced through collinear gluon splitting and  $b$  quark fusion. Shown are the cross sections for  $\sqrt{s} = 7, 10, 14$  TeV.

## A MODEL WITH $\eta_U$ NATURALLY SMALL

The charged scalar in the two Higgs doublet model has couplings to the quarks that (at leading order in the Yukawa matrices) are given by,

$$\mathcal{L}_{\text{charged}} = \eta_U \bar{u}_R g_U d_L S^+ + \eta_D \bar{d}_R g_D u_L S^- + \text{h.c.} \quad (38)$$

We need large CP violating phases to get the fit value of  $\sigma_q$ . For large phases the limits on electric dipole moments suggest that  $|\eta_D \eta_U| \lesssim 10^{-1}$  for charged scalars with mass of a few hundred GeV. Unless the charged scalars are considerably heavier than the neutral ones this bound is expected to hold in the minimal two scalar doublet model if the model is to give the central value for  $h_q$ . If the limits on the electric dipole moments improve then this may become a more serious constraint.

There does not seem to be any acceptable symmetry reason that this product is small and at the same time  $\eta_D$  is enhanced. In order to see that this is the case it is convenient to work in the basis where both  $H$  and  $S$  get a vevs  $v_H$  and  $v_S$  respectively. These can be chosen to be real. Then the charged scalar  $P^+$  is the linear combination of the fields  $h^+$  and  $S^+$ ,

$$P^+ = \frac{v_S h^+ - v_H S^+}{\sqrt{v_H^2 + v_S^2}} \quad (39)$$

Because  $H$  no longer plays a special role we write the couplings of the scalars to the quarks as,

$$\mathcal{L}_Y = \epsilon_H \bar{u} \tilde{g}_U Q_L H + \epsilon_S \bar{u}_R \tilde{g}_U Q_L S \quad (40)$$

$$+ \epsilon'_H \bar{d}_R \tilde{g}_D Q_L H^\dagger + \epsilon'_S \bar{d}_R \tilde{g}_D Q_L S^\dagger + \text{h.c.}$$

Here we are using MFV and taking the quantities that break the flavor symmetry to be  $\tilde{g}_{U/D}$ . These matrices are proportional to the usual Yukawa matrices,

$$g_U = \left( \frac{\epsilon_H v_H + \epsilon_S v_S}{\sqrt{v_H^2 + v_S^2}} \right) \tilde{g}_U, g_D = \left( \frac{\epsilon'_H v_H + \epsilon'_S v_S}{\sqrt{v_H^2 + v_S^2}} \right) \tilde{g}_D. \quad (41)$$

Writing the charged scalar interaction as

$$\mathcal{L}_{\text{charged}} = \eta_U \bar{u}_R g_U d_L P^+ + \eta_D \bar{d}_R g_D u_L P^- + \text{h.c.} \quad (42)$$

we find that,

$$\eta_U = \frac{\epsilon_H v_S - \epsilon_S v_H}{\epsilon_H v_H + \epsilon_S v_S}, \quad \eta_D = \frac{\epsilon'_H v_S - \epsilon'_S v_H}{\epsilon'_H v_H + \epsilon'_S v_S}. \quad (43)$$

One way to get  $\eta_U$  small while  $\eta_D$  is large is to have  $v_H \gg v_S$  so that  $\eta_U \sim -\epsilon_S/\epsilon_H$  and  $\eta_D \sim \epsilon'_H/\epsilon'_S$  and take the corresponding ratios of  $\epsilon$ 's to be small and large respectively. For their product to be small we also need,  $\epsilon_S \epsilon'_H/\epsilon'_S \epsilon_H$  to be small. This is clearly possible, however there doesn't appear to be any symmetry reason behind these choices.

Note that there is an interchange symmetry where  $H \leftrightarrow S$  that forces both  $\eta_U = \eta_D = 0$ . But in the limit of that symmetry,  $P^+ = (h^+ - S^+)/\sqrt{2}$ , and the symmetry's action on  $P^+$  is  $P^+ \rightarrow -P^+$ . Hence there is a stable charged scalar.

The Glashow-Weinberg model [39] where  $H$  couples to the  $u$ -type quarks and  $S$  couples to the  $d$ -type quarks has,  $\epsilon'_H = \epsilon_S = 0$  and so  $\eta_U = v_S/v_H$  and  $\eta_D = -v_H/v_S$ . In this model  $\eta_D \eta_U = -1$ , so when  $\eta_U$  is small  $\eta_D$  is large, but their product cannot be made small even with a tuning of parameters.

In this section we construct a MFV model that has  $\eta_U$  small for a symmetry reason. New scalars that transform under flavour [40] can naturally have a small  $\eta_U$ . Consider a scalar field  $S_8$  that transforms the same way as the Higgs doublet under the gauge group but as  $(\mathbf{1}, \mathbf{8}, \mathbf{1})$  under the flavor group,

$$S_8 \rightarrow V_D S_8 V_D^\dagger. \quad (44)$$

We choose to represent the scalar in terms of the Gell-Mann matrices  $S_8 = S_8^a T^a$  where  $a = 1, \dots, 8$  is a flavour index. The Yukawa couplings are given by

$$\mathcal{L}_Y = \bar{u}_R^i \hat{Y}_U^l (g_D^\dagger)_l^o (T^a)^n_o (g_D)^j_n Q_{Lj} S_8^a, \quad (45)$$

$$+ \bar{d}_R^i (T^a)^m_i (\hat{Y}_D)^j_m Q_{Lj} S_8^{\dagger a} + \text{h.c.}$$

where we have made the flavour indices explicit. We use hat superscripts to distinguish this model's parameters

from the two scalar doublet model. Recall that,

$$\hat{Y}_U^j_i = \hat{\eta}_U g_U^j_i + \hat{\eta}'_U g_U^j_k [(g_U^\dagger)_l^k (g_U)^l_i] + \dots,$$

$$\hat{Y}_D^j_i = \hat{\eta}_D g_D^j_i + \hat{\eta}'_D g_D^j_k [(g_U^\dagger)_l^k (g_U)^l_i] + \dots. \quad (46)$$

The potential is given by

$$V = \frac{\lambda}{4} \left( H^{\dagger i} H_i - \frac{v^2}{2} \right)^2 + 2\hat{m}_1^2 \text{Tr}[S_8^{\dagger i} S_{8i}]$$

$$+ \hat{\lambda}_1 H^{\dagger i} H_i \text{Tr}[S_8^{\dagger j} S_{8j}] + \hat{\lambda}_2 H^{\dagger i} H_j \text{Tr}[S_8^{\dagger j} S_{8i}]$$

$$+ [\hat{\lambda}_3 H^{\dagger i} H^{\dagger j} \text{Tr}[S_{8i}^\dagger S_{8j}] + \hat{\lambda}_4 H^{\dagger i} \text{Tr}[S_8^{\dagger j} S_{8j} S_{8i}]$$

$$+ \hat{\lambda}_5 H^{\dagger i} \text{Tr}[S_8^{\dagger j} S_{8i} S_{8j}] + \text{h.c.}] \quad (47)$$

$$+ \hat{\lambda}_6 \text{Tr}[S_8^{\dagger i} S_{8i} S_8^{\dagger j} S_{8j}] + \hat{\lambda}_7 \text{Tr}[S_8^{\dagger i} S_{8j} S_8^{\dagger j} S_{8i}]$$

$$+ \hat{\lambda}_8 \text{Tr}[S_8^{\dagger i} S_{8i}] \text{Tr}[S_8^{\dagger j} S_{8j}] + \hat{\lambda}_9 \text{Tr}[S_8^{\dagger i} S_{8j}] \text{Tr}[S_8^{\dagger j} S_{8i}]$$

$$+ \hat{\lambda}_{10} \text{Tr}[S_{8i} S_{8j}] \text{Tr}[S_8^{\dagger i} S_8^{\dagger j}] + \hat{\lambda}_{11} \text{Tr}[S_{8i} S_{8j}] \text{Tr}[S_8^{\dagger j} S_8^{\dagger i}].$$

In the potential the index is an SU(2) index and the trace is over the down flavour index. We again rotate the phase of  $S_8$  (relative to  $H$ ) so that the  $\hat{\lambda}_3$  term is real, then the couplings and  $\hat{\lambda}_{4,5}$  and the  $\eta$ 's are in general complex. In the above potential there are no linear terms in  $S_8$  after  $H$  gets its vacuum expectation value and so it is natural for it not to have a vev.

In the potential and the  $\hat{Y}$ 's one can also insert arbitrary numbers of  $g_D g_D^\dagger$  matrices between contractions of a down index. We work in the down basis so that  $g_D = \text{diag}(\sqrt{2}m_d/v, \sqrt{2}m_s/v, \sqrt{2}m_b/v)$ . The interactions in the potential do not change flavour and are suppressed by  $m_b^2/v^2$  so we neglect them.

Keeping just the leading term in,  $\sqrt{2}m_t/v$ , the Wilson coefficient of the effective Hamiltonian as defined in Eq. (10) is

$$C^{\text{NP}}(m_t) = \frac{(\hat{\eta}'_D)^2 (\sqrt{2}m_t/v)^4 \hat{\lambda}_3 m_b^2/6}{\hat{m}_S^4 - \hat{\lambda}_3^2 v^4/4}. \quad (48)$$

where  $\hat{m}_S^2 = \hat{m}_1^2 + (\hat{\lambda}_1 + \hat{\lambda}_2) v^2/4$ . This leads to the mass bound

$$\hat{m}_S^2 \simeq (98 \text{ GeV})^4 |\hat{\eta}'_D|^2 \hat{\lambda}_3 + (174 \text{ GeV})^4 \hat{\lambda}_3^2. \quad (49)$$

in terms of the parameters defined in the potential. The mass spectrum of the new doublet is given by

$$m_{S^\pm}^2 = \hat{m}_S^2 - \hat{\lambda}_2 \frac{v^2}{4},$$

$$m_{S_R^0}^2 = \hat{m}_S^2 + \hat{\lambda}_3 \frac{v^2}{2},$$

$$m_{S_I^0}^2 = \hat{m}_S^2 - \hat{\lambda}_3 \frac{v^2}{2}, \quad (50)$$

We show the masses of the neutral scalars for this model in Fig.7.



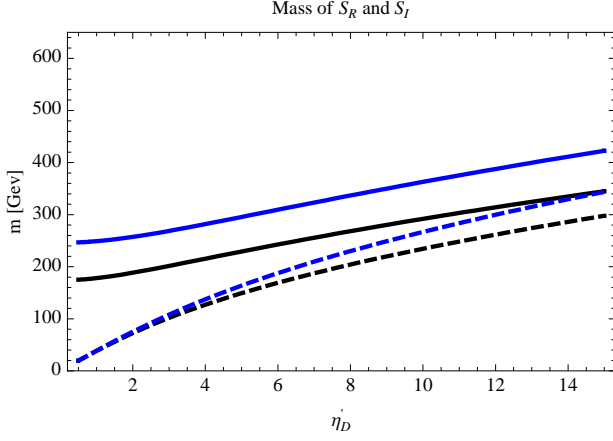


FIG. 7: Mass of  $S_R$  (solid) and  $S_I$  (dashed) as a function of  $\eta_D$  for fixed  $\lambda_3$ . The upper (blue curves) are for  $\lambda_3 = 1$  the lower (black) curves are for  $\lambda_3 = 0.5$ .

This model has eight new scalar doublets. Nevertheless, precision electroweak constraints are satisfied (when the Higgs is fixed to be  $m_h = 96^{+29}_{-24}$  GeV for  $m_S \gtrsim 100$  GeV) when  $|m_I - m_{\pm}| < 50$  GeV [35]. Conversely, custodial SU(2) violation in such a light scalar doublet leading to a positive contribution to  $\Delta T$  can raise the allowed mass of the Higgs in EWP [35, 41].

In this model the coupling constant analogous to  $\eta_U$  is naturally of order  $(m_b/v)^2 \sim 10^{-3}$  and the  $\bar{B} \rightarrow X_S \gamma$  and neutron EDM effects of the model are suppressed as phenomenologically required due to MFV.

The collider phenomenology in this model is very similar to the discussion on the two scalar doublet model. The LEP II constraints allow a larger parameter space due to the smaller mass splitting. The main differences for the Tevatron is that the cross sections we have discussed are increased by an order of magnitude due to the larger flavour representation. Slightly smaller production cross sections through  $b$  quark fusion with low  $p_T$  spectator  $b$  quarks are expected at LHC as the normalization of the Gell Mann matrix decreases the cross section by a factor of three.

We consider speculation on the UV origin of such a  $S_8$  doublet, or other accompanying non flavour singlet doublets that transform under the  $SU(3)_{U_R}$  as an **8** to be premature and beyond the scope of this work.

## CONCLUSIONS

In this paper we discussed the new (i.e., beyond the minimal standard model) physics in the region of parameter space for which the two scalar doublet model with MFV gives the additional contributions to  $B_q - \bar{B}_q$  mixing that are hinted at by the data on flavor physics in the  $B$ -sector. It requires additional light scalars that may be discovered at the Tevatron or LHC. Experimental limits

on electric dipole moments suggest a region of parameter space that can occur naturally in some models where the new doublet of scalars transforms non-trivially under the flavour group. We constructed such a model.

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## APPENDIX: EWP CALCULATIONS

The one loop results for the  $S_8$  model are the same as for the model discussed in [32, 35]. We use the STUVWX parameterization [43] of EWP as the mass scale of the new scalars is  $\sim 100$  GeV. The relevant results in terms of Passarino-Veltman functions [42] with standard definitions<sup>9</sup> are

$$\begin{aligned} \delta\Pi_{WW}(p^2) &= \frac{g_1^2}{2\pi^2} \left[ B_{22}(p^2, m_I^2, m_+^2) + B_{22}(p^2, m_R^2, m_+^2), \right. \\ &\quad \left. - \frac{1}{2}A_0(m_+^2) - \frac{1}{4}A_0(m_R^2) - \frac{1}{4}A_0(m_I^2) \right], \\ \delta\Pi_{ZZ}(p^2) &= \frac{g_1^2}{2\pi^2 c^2} \left[ (1 - 2s^2)^2 \left( B_{22}(p^2, m_+^2, m_+^2) - \frac{1}{2}A_0(m_+^2) \right), \right. \\ &\quad \left. + B_{22}(p^2, m_R^2, m_I^2) - \frac{1}{4}A_0(m_R^2) - \frac{1}{4}A_0(m_I^2) \right], \\ \delta\Pi_{\gamma\gamma}(p^2) &= \frac{2e^2}{\pi^2} \left[ B_{22}(p^2, m_+^2, m_+^2) - \frac{1}{2}A_0(m_+^2) \right], \\ \delta\Pi_{\gamma Z}(p^2) &= \frac{eg_1(1 - 2s^2)}{\pi^2 c} \left[ B_{22}(p^2, m_+^2, m_+^2) - \frac{1}{2}A_0(m_+^2) \right]. \end{aligned}$$

For  $p^2 = 0$  these expressions become

$$\begin{aligned} \delta\Pi_{WW}(0) &= \frac{g_1^2}{8\pi^2} \left( \frac{1}{2}f(m_+, m_R) + \frac{1}{2}f(m_+, m_I) \right), \\ \delta\Pi_{ZZ}(0) &= \frac{g_1^2}{8\pi^2 c^2} \left( \frac{1}{2}f(m_R, m_I) \right), \end{aligned}$$

where

$$f(m_1, m_2) = m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}.$$

<sup>9</sup> With  $c, s$  the cosine and sine of the weak mixing angle.



The derivatives of the vacuum polarizations are

$$\begin{aligned}
\delta\Pi'_{\gamma\gamma}(0) &= -\frac{e^2}{6\pi^2}B_0(0, m_+^2, m_+^2), \\
\delta\Pi'_{\gamma Z}(0) &= -\frac{eg_1(1-2s^2)}{12\pi^2c}B_0(0, m_+^2, m_+^2), \\
\delta\Pi'_{WW}(p^2) &= \frac{g_1^2}{2\pi^2}\left[-\frac{1}{6}\Delta + \frac{\partial b_{22}(p^2, m_I^2, m_+^2)}{\partial p^2}, \right. \\
&\quad \left. + \frac{\partial b_{22}(p^2, m_R^2, m_+^2)}{\partial p^2}\right], \\
\delta\Pi'_{ZZ}(p^2) &= \frac{g_1^2}{2\pi^2c^2}\left[-\frac{1}{12}\Delta + \frac{\partial b_{22}(p^2, m_R^2, m_I^2)}{\partial p^2}, \right. \\
&\quad \left. (1-2s^2)^2\left(-\frac{1}{12}\Delta + \frac{\partial b_{22}(p^2, m_+^2, m_+^2)}{\partial p^2}\right)\right].
\end{aligned}$$

Using these results we can construct the STUVWX parameters with the standard definitions [43]

$$\begin{aligned}
\frac{\alpha S}{4s^2c^2} &= \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right], \\
&\quad -\frac{(c^2 - s^2)}{sc}\delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0), \\
\alpha T &= \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2}, \\
\frac{\alpha U}{4s^2} &= \left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}\right], \\
&\quad -c^2\left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right], \\
&\quad -s^2\delta\Pi'_{\gamma\gamma}(0) - 2sc\delta\Pi'_{Z\gamma}(0), \\
\alpha V &= \delta\Pi'_{ZZ}(M_Z^2) - \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right], \\
\alpha W &= \delta\Pi'_{WW}(M_W^2) - \left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{ZZ}(0)}{M_W^2}\right], \\
\alpha X &= -sc\left[\frac{\delta\Pi_{Z\gamma}(M_Z^2)}{M_Z^2} - \delta\Pi'_{Z\gamma}(0)\right].
\end{aligned}$$

Here  $\Delta$  is the divergence that cancels in the pseudo-observables STUVWX but we note we calculate in dimensional regularization and  $\overline{\text{MS}}$  in  $d = 4 - 2\epsilon$  dimensions. As the number of degrees of freedom in this  $S_8$  model and in the model [32] are the same, we can directly use the detailed fit results on the allowed masses (determined from these formulas) presented in [35]. These results generally allow masses for fixed  $m_h = 96_{-24}^{+29}$  GeV when  $m_S \gtrsim 100$  GeV characterized by  $|m_I - m_\pm| < 50$  GeV for  $S_8$ .

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